incentive compatible

\[
\max \sum_{i \in \mathcal{N}} V_i
\]

\[
\frac{V_i}{\sqrt{|S_i|}}
\]

1. \(i = 1, 2, 3, \ldots\) - \(S_i \cap S_j \neq \emptyset\)
2. \(\text{-legged at the strategy}\)
3. \(\text{number of legs at the strategy}\)

\[
\sum_{V_i \in \text{OPT}} V_i \leq \sqrt{m} \sum_{V_i \in \text{ALG}} V_i
\]

\[
\sum_{V_i \in \text{ALG}} V_i \geq \sqrt{\sum_{V_i \in \text{ALG}} V_i^2} = \sqrt{\sum_{V_i \in \text{ALG}} \frac{V_i^2 |S_i|}{|S_i|}}
\]

\[
\sum_{V_i \in \text{OPT}} V_i \frac{|S_i|}{|S_i|} \leq \sqrt{\sum_{V_i \in \text{OPT}} V_i^2} \sqrt{\sum_{V_i \in \text{OPT}} \frac{|S_i|}{|S_i|}} \leq \sqrt{\sum_{V_i \in \text{OPT}} V_i^2} \sqrt{m}
\]

\[
\sum_{V_i \in \text{ALG}} V_i \frac{|S_i|}{|S_i|} \geq \sum_{V_i \in \text{OPT}} V_i \frac{|S_i|}{|S_i|}
\]

\[
\sum_{V_i \in \text{OPT}} V_i \frac{|S_i|}{|S_i|} \leq \sum_{i \in \text{OPT}} V_i \frac{|S_i|}{|S_i|}
\]

\[
\sum_{i \in \text{OPT}} V_i \frac{|S_i|}{|S_i|} \leq \sum_{i \in \text{OPT}} V_i \frac{|S_i|}{|S_i|}
\]

\[
\sum_{V_i \in \text{OPT}} V_i \frac{|S_i|}{|S_i|} \leq \sum_{i \in \text{OPT}} V_i \frac{|S_i|}{|S_i|}
\]

\[
\sum_{V_i \in \text{OPT}} V_i \frac{|S_i|}{|S_i|} \leq \sum_{i \in \text{OPT}} V_i \frac{|S_i|}{|S_i|}
\]
$$\sum_{j \in \text{OPT}} \frac{V_i^2 |S_j|}{|S_i|} \leq \sum_{j \in \text{OPT}} \frac{V_i^2 |S_j|}{|S_i|}$$

Summing over all players in OPT:

$$\sum_{i \in \text{ALG}} \sum_{j \in \text{OPT}} \frac{V_i^2 |S_j|}{|S_i|} \leq \sum_{i \in \text{ALG}} \frac{V_i^2 |S_i|}{|S_i|}$$

المت: \(\text{OPT} \cap \text{ALG} \neq \emptyset\)

 النظر: \(\text{OPT} \cap \text{ALG} \neq \emptyset\) לכל \(i\) הרציה \(S_i\) בטובות בכל הרציה אחרת.

نظر: \(\text{OPT} \subseteq \text{ALG}\) \(\exists j\) \(\in \text{ALG}\) \(\forall i \in \text{OPT}\) \(V_i > V_j\).

בנוסף, \(\forall i \in \text{OPT}\) \(\exists j\) \(\in \text{ALG}\) \(V_i > V_j\).

כשת נעבור את השיחה:

**digital goods**

שם במרז אתחול מגע ערכ.

$$\sum_{V_i \geq P} V_i$$

הטרנה: לחץ לכל שיחה מותרי \(\sum V_i \geq P\).

$$\alpha \sum_{V_i \geq P} p_i > \sum_{V_i > P} V_i$$

אלגוריתם "קרא" מהו שיחה \(\alpha\)شرط.
Sharing-Cost

The Cost-Sharing problem:

Given a set of goods \( V \) and a set of buyers \( K \), the goal is to allocate the cost of each good \( V_i \) to the buyers in a way that satisfies the following conditions:

1. Each buyer pays at most their valuation of the good.
2. The total cost of all goods is divided among the buyers.

Mathematically, this can be expressed as:

\[
\sum_{i=1}^{K} \frac{C}{K} > C
\]

where \( C \) is the cost of the good, and \( K \) is the number of buyers.

Digital goods

Let \( V \) be a set of digital goods, \( P \) be a set of potential buyers, and \( V_i \) be the valuation of good \( i \) by buyer \( P \).

Define:

\[
V_{max} := \max_i (V_i)
\]

Then, the sharing cost for good \( V_i \) can be defined as:

\[
V_i = V_i = V_{max} - \sum_{i=1}^{K} \frac{C}{K}
\]

Cost-Sharing

Cost-Sharing maximizes the total welfare of the buyers subject to the constraint that each buyer pays at most their valuation of the good. This can be formalized as:

\[
\frac{1}{2} F_1 F_2 \text{ s.t. } F_1, F_2 \text{ are the welfare of the two buyers}
\]

where \( F_1 \) and \( F_2 \) are the welfare functions of the two buyers.

The problem can be solved using linear programming techniques.

References:

The sum of the optimal solution is:

\[ \frac{\min(P_1, P_2)}{OPT} \geq \frac{\min(PK_1, PK_2)}{PK} = \frac{\min(K_1, K_2)}{K} \]

Consequently:

\[ \frac{1}{k} \sum_{i=1}^{k} \min(i, k-i) \Pr \{ \min(i, k-i) = i \} = \frac{1}{k} \sum_{i=1}^{k} \min(i, k-i) \binom{k}{i} 2^{-i} \geq \frac{1}{4} \]

מ.ש.ל.