The Elusive Goal of Workload Characterization

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Abstract

The study and design of computer systems requires good models of the workload to which these systems are subjected. Until recently, the data necessary to build these models—observations from production installations—were not available, especially for parallel computers. Instead, most models were based on assumptions and mathematical attributes that facilitate analysis. Recently a number of supercomputer sites have made accounting data available that make it possible to build realistic workload models. It is not clear, however, how to generalize from specific observations to an abstract model of the workload. This paper presents observations of workloads from several parallel supercomputers and discusses modeling issues that have caused problems for researchers in this area.

1 Introduction

We like to think of building computer systems as a systematic process of engineering—we define requirements, draw designs, analyze their properties, evaluate options, and finally construct a working system. But in many cases the resulting products are sufficiently complex that it makes sense to study them as opaque artifacts, using scientific methodology of observation and measurement. In particular, such observations are necessary to validate (or refute) the assumptions used in the analysis and evaluation. Thus observations made using one generation of systems can be used as a basis for the design and evaluation of the next generation.

A memorable example of such a cycle occurred recently in the field of data communications. Such communications had traditionally been analyzed using Poisson-related models of traffic, which indicated that the variance should smooth out over time and when multiple data sources are combined. But in 1994 Leland and co-workers showed, based on extensive observations and measurements, that this does not happen in practice [27]. Instead, they proposed a fractal traffic model that captures the burstiness of network traffic and leads to more realistic evaluations of required buffer space and other parameters.

Analyzing network traffic was easy, in some sense, because all packets are of equal size and the only characteristic that required measurement and modeling was the arrival process. But if we consider a complete computer system, the problem becomes more complex. For example, a computer program may require a certain amount of memory, CPU time, and I/O, and these resource requirements may be interleaved in various ways during its execution. In addition there are several levels at which we might model the system: we can study the functional units used by a stream of instructions, the subsystems used by a job during its execution, or the requirements of jobs submitted to the system over time. Each of these scales is relevant for the design and evaluation of different parts of the system: the CPU, the hardware configuration, or the operating system.

In this paper we focus on characterization of the workload on a parallel system, from an operating system perspective (as was done in [12, 38]). Thus we investigate means to characterize the stream of parallel jobs submitted to the system, their resource requirements, and their behavior. We show that such characterization must contend with various problems; some can be addressed easily, but others seem to be hard to avoid.

1.1 Definitions

We use *size* to refer to the number of processors used by a parallel program (as shorthand for "cluster size", "partition size", or "degree of parallelism"). We use *duration* to refer to the span of time starting when a job commences execution and ending when it terminates (otherwise known as "runtime" or "lifetime"; it is also the same as "response time" if the job did not have to wait). The product of these two sizes, which represents the total resources (in CPU-seconds) used by the job, is called its *area*.

2 Modeling

There are two common ways to use a measured workload to analyze or evaluate a system design: (1) use the traced workload directly to drive a simulation, or (2) create a model from the trace and use the model for either analysis or simulation. For example, trace-driven simulations based on large address traces are often used to evaluate cache designs [23, 22]. But models of how applications traverse their address space have also been proposed, and provide interesting insights into program behavior [36, 37].

The advantage of using a trace directly is that it is the most "real" test of the system; the workload reflects a real workload precisely, with all its complexities, even if they are not known to the person performing the analysis.

The drawback is that the trace reflects a specific workload, and there is always the question of whether the results generalize to other systems. In particular, there are cases where the workload depends on the system configuration, and therefore a given workload is not necessarily representative of workloads on systems with other configurations. Obviously, this makes the comparison of different configurations problematic. In addition, traces are often misleading if we have incomplete information about the circumstances when they were collected. For example, workload traces often contain intervals when the machine was down or part of it was dedicated to a specific project, but this information may not be available.

Workload models have a number of advantages over traces.

- The modeler has full knowledge of workload characteristics. For example, it is easy to know which workload parameters are correlated with each other because this information is part of the model.
- It is possible to change model parameters one at a time, in order to investigate the influence of each one, while keeping other parameters constant. This allows for direct measurement of system sensitivity to the different parameters. It is also possible to select model parameters that are expected to match the specific workload at a given site.

  In general it is not possible to manipulate traces in this way, and even when it is possible, it can be problematic. For example, it is common practice to increase the modeled load on a system by reducing the average interarrival time. But this practice has the undesirable consequence of shrinking the daily load cycle as well. With a workload model, we can control the load independent of the daily cycle.
- A model is not affected by policies and constraints that are particular to the site where a trace was recorded. For example, if a site configures its NQS queues with a maximum allowed duration of 4 hours, it forces users to break long jobs into multiple short jobs. Thus, the observed distribution of durations will be different from the "natural" distribution users would have generated under a different policy.
- Logs may be polluted by bogus data. For example, a trace may include records of jobs that were killed because they exceeded their resource bounds. Such jobs impose a transient load on the system, and influence the arrival process. However, they may be replicated a number of times before completing successfully, and only the successful run represents "real" work. In a model, such jobs can be avoided (but they can also be modeled explicitly if so desired).
- Finally, modeling increases our understanding, and can lead to new designs based on this understanding. For example, identifying the repetitive nature of job submittal can be used for learning about job requirements from history. One can design a resource management policy that is parameterized by a workload model, and use measured values for the local workload to tune the policy.

The main problem with models, as with traces, is that of representativeness. That is, to what degree does the model represent the workload that the system will encounter in practice?

Once we decide to construct a model, the chief concern is what degree of detail to include. As noted above, each job is composed of procedures that are built of instructions, and these interact with the computer at different levels. One option is to model these levels explicitly, creating a hierarchy of interlocked models for the different levels [5, 2]. This has the obvious advantage of conveying a full and detailed picture of the structure of the workload. In fact, it is possible to create a whole spectrum of models spanning the range from condensed rudimentary models to direct use of a detailed trace.

For example, the sizes of a sequence of jobs need not be modeled independently. Rather, they can be derived from a lower-level model of the jobs' structures [14]. Hence the combined model will be useful both for evaluating systems in which jobs are executed on predefined partitions, and for evaluating systems in which the partition size is defined at runtime to reflect the current load and the specific requirements of jobs.

The drawback of this approach is that as more detailed levels are added, the complexity of the model increases. This is detrimental for two reasons. First, more detailed traces are needed in order to create the lower levels of the model. Second, it is commonly the case that there is wider diversity at lower levels. For example, there may be many jobs that use 32 nodes, but at a finer detail, some of them are coded as data parallel with serial and parallel phases, whereas others are written with MPI in an SPMD style. Creating a representative model that captures this diversity is hard, and possibly arbitrary decisions regarding the relative weight of the various options have to be made.

In this paper we concentrate on *single-level* models, which may be viewed as the top level in a hierarchical set of models [14]. Even with this restriction, difficulties abound.

3 Statistics

The whole point of a model is to create a concise representation of the observed workload. Thus it is desirable to derive a small set of numbers or a simple function that characterizes each aspect of the workload. If we cannot do this, we are left with the original data as the best representation.

Any time we reduce a set of observations to a small set of numbers, the result is called a summary statistic. These fall into three families (See Jain's treatise for more details [19, chap. 12]):

Moment-based: The kth moment of a sequence $x_1, x_2, \ldots x_n$ of observations is defined by $m_k = \frac{1}{n}\sum x_i^k$. Important statistics derived from moments include

- The mean, which represents the "center of gravity" of a set of observations: $\bar{x} = \frac{1}{n}\sum x_i$.
- The standard deviation, which gives an indication regarding the degree to which the observations are spread out around the mean: $s = \sqrt{\frac{1}{n-1}\sum (x_i - \bar{x})^2}$.
- The coefficient of variation, which is a normalized version of standard deviation: $cv = s/\bar{x}$.

Percentile-based: Percentiles are the values that appear in certain positions in the sorted sequence of observations, where the position is specified as a percentage of the total sequence length. Important statistics derived from percentiles include

- The median, which represents the center of the sequence: it is the 50th percentile, which means that half of the observations are smaller and half are larger.
- Quartiles (the 25, 50, and 75 percentiles) and deciles (percentiles that are multiples of 10). These give an indication of the shape of the distribution.
- The semi-interquartile range (SIQR), which gives an indication of the spread around the median. It is defined as the average of the distances from the median to the 25 percentile and to the 75 percentile.

Mode-based. The mode of a sequence of observations is the most common value observed. This statistic is obviously necessary when the values are not numerical, for example when they are user names. It is also useful for distributions that have strong discrete components, as may indeed happen in practice (e.g. see Figs. 2 and 4 below).

The use of moments (mean, variance, etc.) as summary statistics is common because they are easy to calculate and most people have some intuition about their meaning (although this intuition starts to wane somewhere between skew, the third moment, and kurtosis, the fourth).

Unfortunately, many researchers are in the habit of reporting these statistics for non-symmetric distributions. At best, this practice is misleading, since the reader's intuition about the distributions will be wrong. At worst, calculated moments are meaningless values.

There are two reasons moments may fail to summarize a distribution:

- Nonconvergence: For some distributions, the moments of a sample do not converge on the moments of the population, regardless of sample size. An example that is common in computer science is the Pareto distribution[1] [21]. For Pareto distributions with shape parameter a less than 1, the mean and other moments are infinite. Of course, any finite sample will have finite moments, so convergence is not possible. A warning sign of nonconvergence is the failure to achieve sample invariance as sample size increases.

[1] The CDF of the Pareto distribution is $F(x) = 1 - x^{-a}$ and the pdf is $f(x) = ax^{-(a+1)}$, where the shape parameter satisfies $a > 0$.

- Noise: Even for symmetric distributions, moment statistics are not robust in the presence of noise. A small number of outliers can have a large effect on mean and variance. Higher moments are even less reliable [34].

  This effect is particularly problematic for long-tailed distributions, where it may be impossible to distinguish erroneous data from correct but outlying values. In this case even the mean should be used with caution, and variance should probably be avoided altogether.

3.1 The moments problem: an example

The following example is taken from a real data set, the durations of jobs submitted to the IBM SP2 at the Cornell Theory Center [18]. The data set contains the start and end time for each job that was submitted from June 6 to December 2, 1995 (in seconds since the beginning of the epoch). We calculated the (wall-clock) duration of each job by subtracting the start time from the end time. Of the 50866 records, two were corrupted such that the duration could not be calculated.

There are a few jobs that appear to have very long durations (one job appears to have run for over a month). It is possible that these outliers are caused by corruption of the data set, but it is also possible that these jobs experienced an unusual condition during their execution that caused them to be swapped out or held for a long time. The accounting data do not provide this information.

So how should we deal with these outliers? It might be tempting to remove possible errors by omitting some fraction of the longest jobs from the data. The following table shows the result, for a range of omitted data. The coefficient of variation (CV) is the ratio of the standard deviation to the mean.

Rec's omitted		statistic (% change)		
	(% of total)	mean [sec]	CV	median [sec]
0	(0%)	9371	3.1	552
5	(0.01%)	9177 (-2.1%)	2.2 (-29%)	551 (-0.2%)
10	(0.02%)	9094 (-3.0%)	2.0 (-35%)	551 (-0.2%)
20	(0.04%)	9023 (-3.7%)	1.9 (-39%)	551 (-0.2%)
40	(0.08%)	8941 (-4.6%)	1.9 (-39%)	550 (-0.4%)
80	(0.16%)	8834 (-5.7%)	1.8 (-42%)	549 (-0.5%)
160	(0.31%)	8704 (-7.1%)	1.8 (-42%)	546 (-1.1%)

As we discard suspect values from the data set, the values of the moments change considerably. Since it is impossible to say where to draw the line—that is, which data are legitimate and which bogus—it is impossible to say meaningfully what the mean and variance of this distribution are. The median and other order statistics are not as sensitive to the presence of outliers.

A second example makes the same point more emphatically. From the same data set, we calculated the area of each job (the product of its wall clock duration and the number of processors it allocated). Again, there are a few large values that dominate the calculation of the moments. The following table shows what happens as we try to eliminate the outlying data.

Rec's omitted		statistic (% change)		
	(% of total)	mean [sec]	CV	median [sec]
0	(0%)	88,586	29.9	2672
5	(0.01%)	72,736 (-18%)	7.1 (-76%)	2672 (-.00%)
10	(0.02%)	69,638 (-21%)	5.8 (-81%)	2670 (-.00%)
20	(0.04%)	66,632 (-25%)	5.1 (-83%)	2666 (-.00%)
40	(0.08%)	62,265 (-30%)	4.5 (-85%)	2656 (-.00%)
80	(0.16%)	58,515 (-34%)	4.0 (-87%)	2654 (-.00%)
160	(0.31%)	53,563 (-40%)	3.8 (-87%)	2633 (-.01%)

Depending on how many data are discarded, the values of the mean and CV vary wildly. If only 5 values (of 50864) are discarded, the CV drops by a factor of four! Thus a few errors (or the presence of a few unusually big jobs) can have an enormous effect on the estimated moments of the distribution. The median, on the other hand, is almost unaffected until a large fraction of the data is omitted.

As mentioned above, one of the warning signs of non-convergence is the failure to achieve sample invariance. A simple test of sample invariance is to partition the data set into even- and odd-numbered records and compare the summary statistics of the two partitions. The following table shows the results for the duration data set:

Partition	mean [sec]	CV	median [sec]
even records	9450	3.6	543
odd records	9292	2.5	560

The mean and median are reasonably sample invariant (the means differ by only 2%; the medians by 3%). But the CV's differ by 44%, and the higher moments are probably even worse. The results are even more pronounced for the area:

Partition	mean [p·s]	CV	median [p·s]
even records	97383	37.5	2688
odd records	79789	10.1	2656

Because the moments are sensitive to the presence of outliers, they should not be used to summarize long-tailed distributions. If they are, they should be tested for sample invariance and reported with appropriate confidence intervals, keeping in mind that the usual method of calculating confidence intervals (using a t-distribution and calculated standard errors) is based on the assumption of symmetric distributions, and is not appropriate for long-tailed distributions.

3.2 Robust estimation

A summary statistic that is not excessively perturbed by the presence of a small number of outliers is said to be *robust*. In general, order statistics (median and other percentiles) are more robust than moments [28]. The most common order statistic is the median (50th percentile), but other percentiles are also used. For example, Jain recommends the use of percentiles and the SIQR to summarize the variability of distributions that are unbounded and either multimodal or asymmetric [19, Figure 12.4].

The importance of using percentile-based statistics rather than moment-based statistics in workload modeling cannot be over-emphasized. Most real-life distributions encountered in this domain are asymmetric and long-tailed. For example, it is well known that the distribution of process durations has a CV that is larger than 1 (in some cases, much larger). More than 20 years ago, Lazowska showed that models based on a hyperexponential distribution with matching moments lead to incorrect results [25]. Instead, percentiles should be used.

The question is then how to build a model based on percentiles. One possibility is to find a simple curve that fits the observed cumulative distribution function (CDF) and then use the parameters of the fitted curve as summary statistics. The CDF is the inverse of the percentile function; we can find the nth percentile of a distribution by inverting the CDF. Thus, we can think of the CDF itself as a robust summary statistic.

3.3 Log-uniform distributions

Figure 1 shows the CDFs of the two distributions from Section 3.1 (the durations and areas of jobs on the CTC SP2). In both cases, the CDF is approximately linear in log space, implying a uniform distribution. Thus, we can use a *log-uniform* model to summarize these distributions. The two parameters of this model are the lower and upper bounds t_{min} (where the CDF is 0) and t_{max} (where it reaches 1). The probability density function (pdf) for this distribution is

$$pdf_L(t) = \Pr(t \le L < t+dt) = \frac{1}{\kappa t}, \quad t_{min} \le t \le t_{max} \tag{1}$$

where L is a random variable representing the duration of a job, and $\kappa = \ln t_{max} - \ln t_{min}$ is a constant. The mean of the distribution is

$$E[L] = \frac{1}{\kappa}(t_{max} - t_{min}) \tag{2}$$

To derive the median of the model, we use an alternate form for the distribution, based on the two parameters β_0 and β_1, which are the intercept and slope of the cumulative distribution function:

$$CDF_L(t) = \Pr(L \le t) = \beta_0 + \beta_1 \ln t, \quad t_{min} \le t \le t_{max} \tag{3}$$

The upper and lower bounds in terms of the alternate parameters are $t_{min} = e^{-\beta_0/\beta_1}$ and $t_{max} = e^{(1.0-\beta_0)/\beta_1}$. Now we can find the median by setting the CDF to 0.5 and solving for t, which gives

$$t_{med} = e^{(0.5-\beta_0)/\beta_1} = \sqrt{t_{min} t_{max}} \tag{4}$$

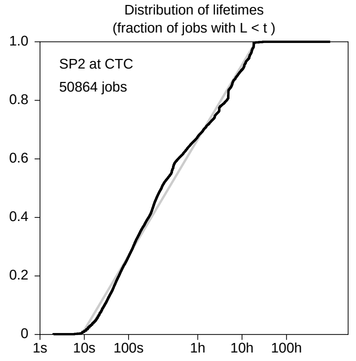
The alternate form of the model is also useful for estimating the parameters of an observed distribution by a least-squares fit. The following table shows the estimated parameters for the two distributions in the example:

Distribution	β_0 (intercept)	β_1 (slope)	R^2
duration	-0.240	0.111	.993
area	-0.304	0.102	.979

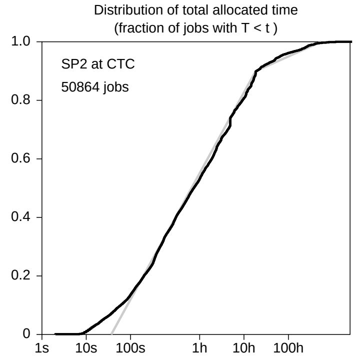
For the duration distribution, the model is a very good fit (the gray line in Figure 1(a) shows the fitted line). The model deviates from the observed distribution for a small number of very long and very short jobs. But this deviation affects very few jobs (roughly 0.3% of jobs fall below the lower bound of the model, and 0.4% exceed the upper bound).

The straight-line model does not fit the area distribution as well, as is reflected in the lower R^2 value. In this case, a better model is a *multi-stage log-uniform* model, which is piecewise linear in log space. The gray line in Figure 1(b) shows a two-stage model we fitted by hand. This model fits the observed distribution well, except for a few short jobs. That deviation is probably not important; the only effect is that some jobs that were expected to run for 30 seconds run only 10 seconds instead.

Based on the fitted distributions, the summary statistics are:



(a)



(b)

Figure 1: *(a) The distribution of durations, and (b) the distribution of areas (the product of duration and size). The gray lines show the models used to summarize each distribution.*

Distribution	mean	CV	median
duration (observed)	9371	3.1	552
duration (model)	7884	1.9	786
area (observed)	88586	29.9	2672
area (two-stage model)	64306	3.6	2361

In each case, the summary statistics of the model are significantly different from those of the observed distribution. Nevertheless, because the model is a good fit for the distribution, we believe it describes the behavior of the observed system reasonably well. Thus, if we were asked to report the CV of the distribution of areas, we would rather say 3.6 (the CV of the fitted model) than, "Something less than 29.9, depending on how many of the data turn out to be bogus."

In prior work, Downey used these techniques to summarize the log-uniform distributions that are common on batch systems [8], and Harchol-Balter and Downey used a similar technique to summarize the Pareto distributions that are common on sequential interactive systems [17].

3.4 Goodness of fit

The above description is based on the eyeball method: we transformed the CDF into log space, observed that the result looked like a straight line, and decided to use this as our model. We *think* this is a reasonable model, that captures the essence of the workload. But does it really?

The main advantage of the log-uniform model is its simplicity: only two parameters are needed. But we might opt for a more complex model that is more accurate. For example, in the case of the distribution of area, we chose to use a two-stage model, which requires four parameters. Hyper-exponential distributions, using three parameters, have also been popular for modeling distributions with high variability. In addition they have the advantage that it is easy to tailor such distributions with a given CV. Alternatively, we might choose the sum of two Gamma distributions, as has been proposed by Lublin [31]. This model uses five parameters and seems to provide an even better fit to the measurements.

There are more formal techniques for evaluating the goodness of fit of these models [24, chap. 15], including the chi-square test and the Kolmogorov-Smirnov test. In the chi-square test, the range of sample values is divided into intervals such that each has at least 5 samples. Then the expected number of samples in each interval is computed, based on the hypothesized distribution function. If the deviation of the actual number of samples from the expected number is sufficiently small, the hypothesis is accepted.

The Kolmogorov-Smirnov test is even simpler — it bases the decision on the maximal difference between the CDF of the hypothesized distribution and the CDF of the actual sample values. However, this test is only valid for continuous distributions.

The problem with applying these methods to workload characterization is that they typically fail. Most parametric statistical methods are based on the assumption that samples are drawn from a population with a known distribution, and that as the sample size increases, the sample distribution converges on the population distribution. Since the sample sizes in the traces are so large, they are expected to match the model distribution with high accuracy. In practice they do not, because the underlying assumption—that the population distribution is known—is false. There is no reason to believe that the distribution of durations really is log-uniform, or the sum of two Gammas, or any other

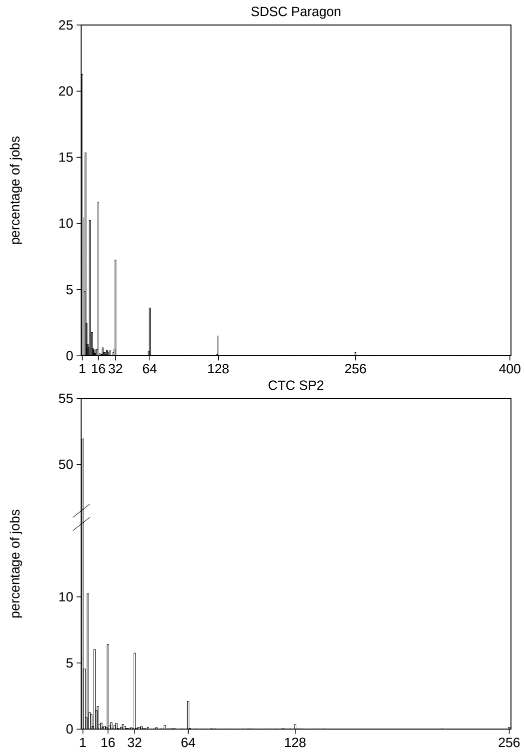


Figure 2: *The observed distribution of sizes typically has many small jobs, and strong discrete components at powers of two.*

simple functional form. Rather, we use these models because they describe the real data concisely, and with sufficient accuracy.

Of course, the question remains, "How much accuracy is sufficient?" Ultimately, the only answer is to see whether the results from the model match results in the real world. Barring that, a useful intermediate step is to compare results from the model with results from workload traces. Lo et al. have done this recently for parallel job scheduling strategies, and found encouraging results—at least for some strategies, the results of trace-driven simulations are consistent with most workload models, even very simple ones [30]. Thus it seems likely that these results will apply to real systems.

3.5 Distributions with discrete components

As noted above, a concise way to describe a distribution is with a mathematical model. But in some cases the trace data indicate that the distribution has discrete components, and may not be amenable to a concise description. One must then decide whether to ignore such discrete components, and if not, whether their exact locations and sizes are random or an inherent characteristic of the workload.

A good example is the distribution of the sizes of parallel jobs. Numerous traces show that this distribution has two salient characteristics: most of the weight is at low values, and strong discrete components appear at powers of two (two examples are given in Figure 2). This result is significant because jobs using power-of-two partitions are easier to pack, and small jobs may be used to fill in holes between larger jobs [6, 13]. Thus including or omitting these features has a significant impact on the performance of the modeled system.

4 Weights

In the previous section we espoused the value of characterizing workloads by using distributions rather than just moments. However, care must be taken in collecting the data used to characterize the distributions. In particular, it is important to give appropriate weights to the different jobs in the workload. Many prior studies have sinned by giving all jobs equal weights, and produced misleading results.

We motivate this section with an example from the airline industry. Airlines often report statistics about the number of empty seats on their planes, and passengers are often surprised by the numbers. For example, the airline might report (truthfully) that 60% of their flights are less than half full, and only 5% are full. At the same time, passengers might observe that most flights are full and very few are less than half full. This discrepancy is sometimes called the "observer's paradox," because it is a result of the fact that more passengers (observers) travel on the full planes than on the empty ones. To determine the probability that a passenger sees a full flight, it is necessary to weight the airline's statistics by the number of passengers on each flight.

Applying this insight to parallel workloads, there are three ways we might weight the distribution of a job attribute, each of which is useful in a different context:

- jobs with equal weights
- jobs weighed by duration
- jobs weighed by area (i.e. by duration and size)

Distributions in which jobs have equal weights are useful when studying certain characteristics of the workload as a whole. For example: what percentage of jobs are interactive (meaning that they

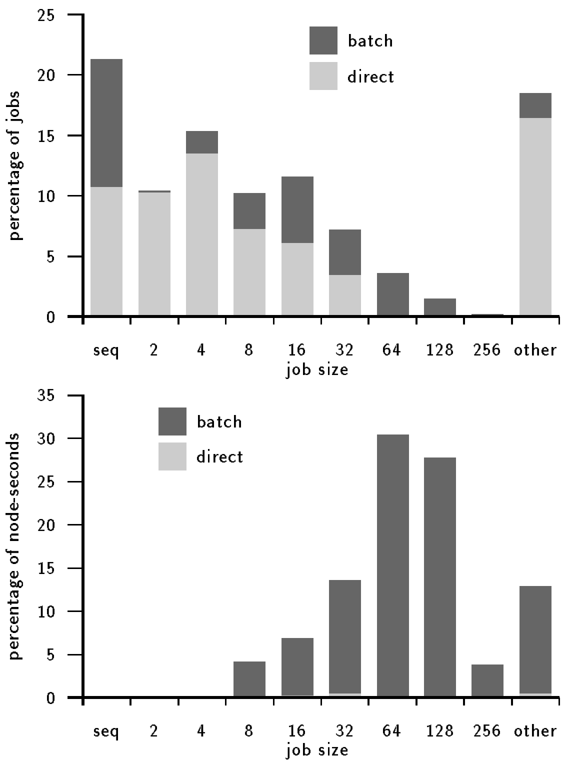


Figure 3: *When duration is used as a weighting factor, rather than giving jobs equal weights, the distribution changes considerably (data from SDSC Paragon).*

complete within a few seconds and a human is probably waiting for them)? To answer this question we can use the distribution of job durations, where jobs have equal weights.

Distributions in which jobs are weighted by duration are useful for investigating per-job resource usage. For example, if you want to know the distribution of job sizes at the time of a new arrival, you should weight each job by its duration, since long-running jobs are more likely to be observed than short-running ones.

The difference between the raw distribution and the weighted distribution may be dramatic, as illustrated in Figure 3. The top distribution, with equal weights, shows that most arriving jobs are small: 50% use 4 nodes or less, and many of them are *direct*, meaning that they are submitted directly in an interactive manner. In the bottom distribution jobs are weighted by their duration. This distribution implies that an arriving job expects to see a few large jobs, since more than 50% of node-seconds are for 64 and 128-node batch jobs. Based on this insight, we conclude that it may be desirable to configure a machine with a large (256–512 node) batch partition, and a small (maybe 32-node) interactive partition, in order to provide responsiveness for interactive users and good throughput for batch.

It should be noted, however, that this recommendation is based on this particular workload, in which hardly any direct jobs use more than 32 nodes. It is not clear, though, whether this limit reflects the desires of users, in which case it makes sense to accommodate it, or whether it is due to the policies in force at SDSC when the trace was collected.

The third weighting scheme, where jobs are weighted by their area, is useful when studying per-processor resource usage. For example, one might want to know the available memory expected on a node that is running one job. Looking at the top distribution in Figure 4, we see that many jobs use a small amount of memory, but that does not answer the question, because many of the jobs that use large amounts of memory are also large, long-running jobs. Because the probability of being observed on a given node depends on duration and size, the correct answer comes from the bottom plot in Figure 4, where jobs are weighted by area (the product of duration and size). Using either of the top two plots, where jobs have equal weights or are weighed only by their duration, leads to overly optimistic predictions.

5 Correlations

Having addressed the issue of how to represent each attribute of the workload in isolation, we now turn to the question of whether these attributes are independent. In fact, we find that they are not, and that there are potentially meaningful correlations between the different attributes of a job. Such correlations should be included in the workload model.

5.1 Are Large Jobs Longer?

We illustrate this point by looking into the correlation between the size and the duration of parallel jobs. Two common assumptions that have appeared in the literature are that size and duration are independent of each other or inversely correlated (that is, larger jobs run for less time on average) [32, 29]. As these assumptions affect the results of evaluations of scheduling policies, it is useful to check whether they are born out by measurements.

The most common way to measure such correlations is with the Pearson product moment correla-

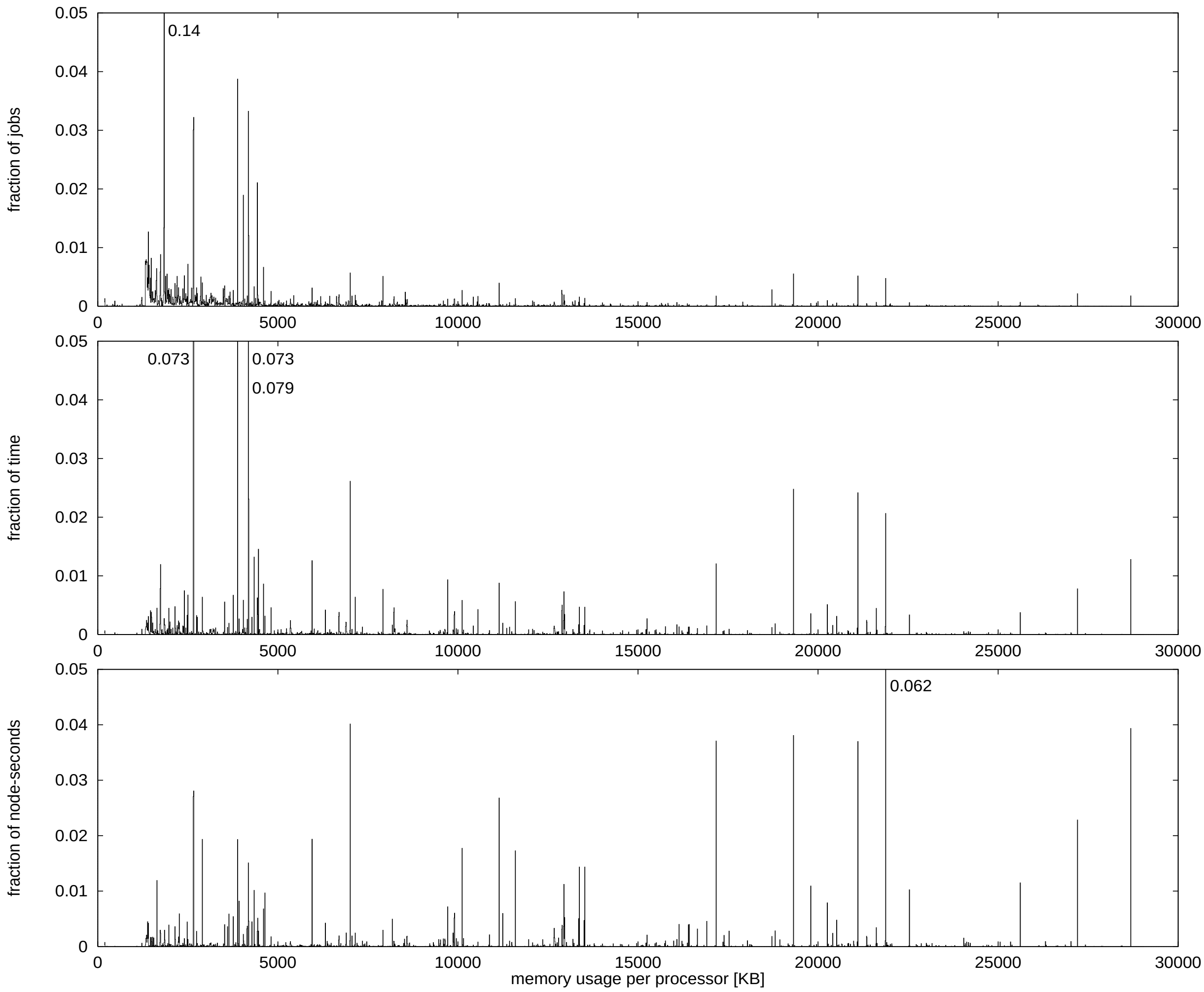


Figure 4: *The distribution of per-processor memory usage on the LANL CM-5 (from [9]), using buckets of 10 KB. In the top plot, all jobs have equal weight. In the middle, jobs are weighted according to their duration. The bottom plot shows the distribution for individual processors, which is equivalent to weighting the jobs according to their area (the product of duration and size).*

tion coefficient, r, which is defined as the covariance divided by both standard deviations:

$$r = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2}\sqrt{\sum(y_i - \bar{y})^2}}$$

The range of absolute values is from 0, indicating no correlation, to 1, indicating perfect correlation [24, chap. 18]). Negative values indicate inverse correlations. For the SDSC data, Pearson's r is 0.24, indicating a weak correlation.

Of course, one problem with this computation is that it suffers from the lack of robustness discussed in Section 3. A good solution is to compute the correlation of x'_i and y'_i, where $x'_i = \log(x_i)$, and $y'_i = \log(y_i)$. By compressing the long tail of the duration distribution, the logarithmic transformation makes the moments more meaningful.

Another problem with Pearson's r is that it is based on the assumption that the relationship between the variables is linear. This assumption does not apply to the workloads we have observed. Figure 5(a) shows average durations for jobs with different sizes. There is no clear relationship; some job sizes have very high average durations, and others have low average durations. Moreover, the different data points represent different numbers of jobs (recall the distribution of sizes shown in Figure 2). Thus an outlier with a unique size shows up much more than the same value in a popular size, where it is averaged with many other jobs.

Figure 5(b) shows the relationship more clearly.

(a)

(b)

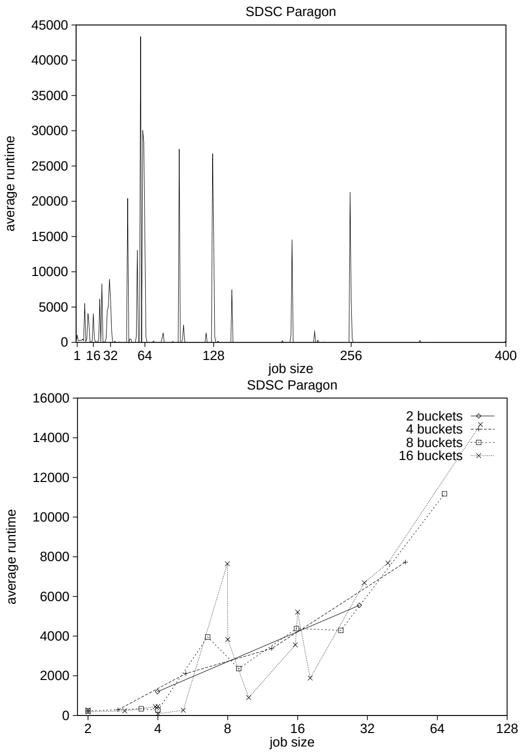


Figure 5: *(a) There is no clear direct relation between average duration and size. (b) By distributing all jobs into equal sized buckets according to size, a correlation between size and duration is observed.*

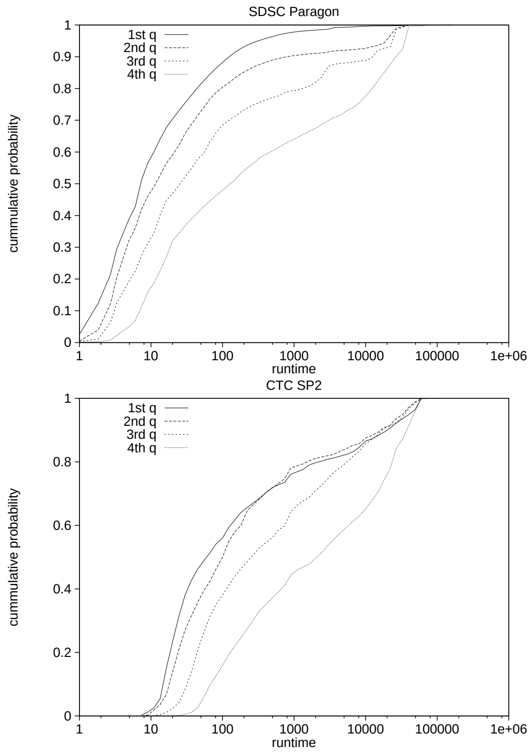


Figure 6: *CDFs of distribution of durations for jobs in four buckets according to size.*

We grouped the jobs into buckets according to their size. The first bucket contains the smallest jobs, the second contains the next smallest, etc. For each bucket we plotted a single point, drawn at the average size and average duration for the jobs in that bucket. The figure shows graphs using 2, 4, 8 and 16 buckets. Using fewer buckets yields fewer data, but shows the trend most clearly; using more buckets yields a more detailed but noisier line. Overall there is a definite trend from lower-left to upper-right, indicating that larger jobs tend to run longer. This contradicts the assumptions commonly made in the literature.

Additional evidence is obtained by plotting the distributions of durations for jobs in the different buckets. Figure 6 shows CDFs for 4 buckets, using the SDSC and CTC data. Clearly jobs with different sizes have different distributions of durations. For small jobs (with few nodes), the weight of the distribution is at short durations. For large jobs (with many nodes), the weight is at long durations. But in all cases the distributions span the whole range from seconds to hours.

This analysis suggests several ways we might generate a random workload with realistic correlations between job attributes:

- We can calculate Pearson's r for an observed workload and then generate a random workload with the same coefficient of correlation.
- We can divide a workload trace into buckets according to size, user name, and other attributes, and find the distribution of durations for each bucket. Then for a given size and user name we can choose a random duration from the appropriate distribution.

The advantage of the first approach is that it involves a minimal number of parameters, and that it smooths out unpredictable variations that are likely to be site-specific, as in Figure 5(a). The disadvantage is that Pearson's r may understate the degree of correlation.

The advantage of the second approach is that it allows for a tradeoff between accuracy and complexity. We can choose to use more attributes and smaller buckets, and create more detailed models, at the cost of more parameters; for example, Jann et al. did so in their work, and essentially produced a separate model of the distribution of durations for each bucket of sizes [20]. On the other hand, we can create a simple parameterized model, with a parameter that depends on the bucket; For example, Feitelson used this approach to create a model of duration that correlates with size, by using a hyperexponential distribution where the probability of choosing each branch depends on the size [10].

5.2 Correlation as a Source of Information

Another advantage of the second approach is that it captures variations that might be significant for evaluating scheduling policies. At most sites there is significant local uniformity within the generally high variability of the workload. That is, while it may be impossible to predict job durations in general, it may be significantly easier if other information about the job (size, user name, etc.) is known.

To measure the usefulness of this information, we can use the coefficient of determination, *CD*, which measures the amount of variation in a dependent variable that can be accounted for using an independent variable as a predictor.

As an example, we divided the data from SDSC and CTC into categories according to size, p, and calculated the mean in each category, μ_p, and the variance around that mean. The weighted average of the variances, $\tilde{\sigma}^2$, indicates the width of the distribution of durations, *taking size into account.*

$$\tilde{\sigma}^2 = \frac{1}{N} \sum (x_i - \mu_{p_i})^2 \tag{5}$$

where p_i is the size of the ith job, and N is the total number of jobs.

The ratio of this variance to the original variance indicates what fraction of the variation remains, given that we know the cluster size. The resulting coefficient of determination is

$$CD^2 = 1 - \left(\frac{\tilde{\sigma}^2}{\sigma^2}\right) \tag{6}$$

where σ^2 is the conventional variance. The value of CD^2 is comparable to the R^2 value used to evaluate the goodness-of-fit of a linear regression. It has the same range as Pearson's r, with 0 indicating no relationship between the variables and 1 indicating that it is possible to predict one of the variables perfectly, given the value of the other.

As an example, for the CTC data Pearson's r (calculated with the log transform) is 0.0038, which would seem to indicate no significant correlation. The value of *CD* is 0.21, indicating that the size does contribute some information about expected durations. For the SDSC data, Pearson's r is 0.25 and *CD* is 0.38.

These calculations, and other evidence of local predictability [16, 35], suggest that simple measurements of correlation, and models based on them, omit important workload characteristics. Although we have suggested some techniques for measuring these characteristics and incorporating them into a model, it is still not clear how to distinguish significant, general characteristics that should be included from site-specific peculiarities that should be ignored.

6 Time Scale

So far we have treated workload traces as if they we monolithic; however, it is often the case that workloads change over time, due to

- Evolution as users learn to use a new system and exploit its capabilities [18],
- Adaptation as schedulers and allocation policies are changed by a system's administrators [11], and
- Random fluctuations caused by people's work schedules, deadlines, etc.

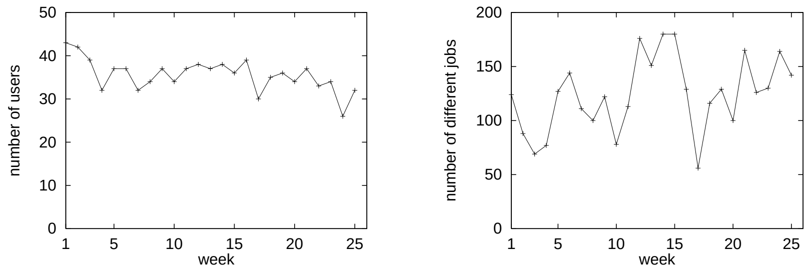
Thus the created model depends on the length of the trace and the exact period during which it was recorded. Should we average over a whole year or only over a week? And if we use only one week, which week should it be?

6.1 Uniformity in short time scales

If we were to measure the degree of variability in the workload as a function of the time interval under study, this would be a monotonically increasing function[2]. When we look at relatively short time intervals, say a week, we find that the variability is rather low: few users are active, few distinct applications are executed, etc. But as the weeks accumulate, we see that each one is significantly different from the others. As an example, Figure 7 shows a set of histograms of job sizes in 25 consecutive weeks at the beginning of 1996, taken from the LANL CM-5. Both the absolute numbers of jobs and the distribution of sizes change considerably from week to week.

[2] This is not meant as a precise statistical claim, e.g. that the variance of the durations of all jobs is monotonically increasing. Instead, it refers to the richness or cardinality of the sets of possible parameter values, such as the set of users, the set of applications, and the set of durations.

Figure 9: *Number of active users and number of distinct applications executed in successive weeks.*



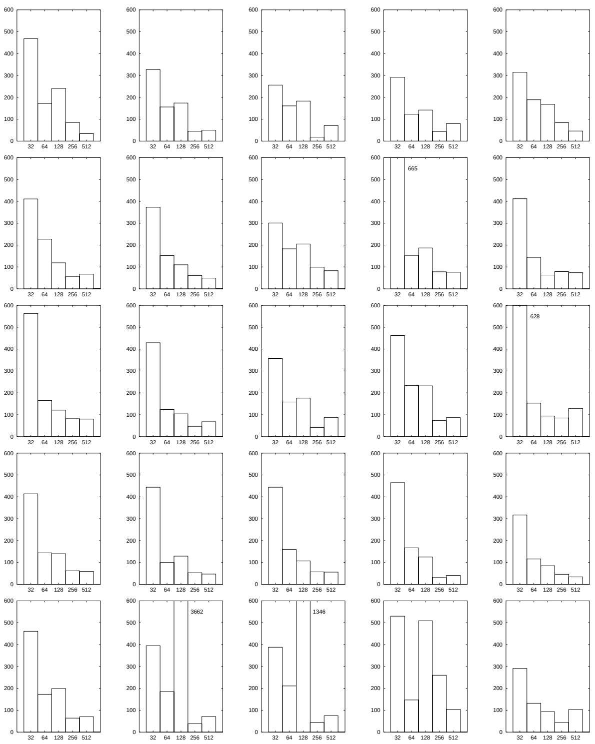


Figure 7: *Histograms of job sizes in 25 consecutive weeks on the LANL CM-5. The five bars in each histogram represent jobs using 32, 64, 128, 256, and 512 processors. The scale is up to 600 jobs, but in some cases, higher values were recorded.*

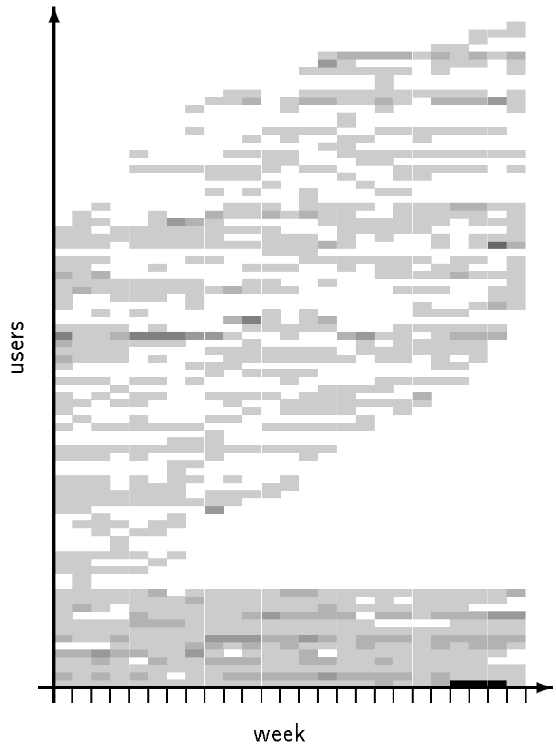


Figure 8: *Gantt chart of user activity in successive weeks. Gray level indicates number of jobs run.*

One source of variability over time is that different users are active during different intervals. Figure 8 is a Gantt chart of user activity in the same 25 weeks shown in Figure 7. Each user is represented by a horizontal line, and the gray shading indicates that user's level of activity (measured in number of jobs submitted). The users are divided into two groups: at the bottom are those that were active for more than 80% of the time; there were 13 such users. Above them are the more sporadic users, ordered according to the center of the period of their activity. Obviously, the community of users shifts with time.

Since the behavior of different users varies widely, and the community of active users changes over time, we expect that the overall workload is more predictable during a short interval (a week) than during a long interval (a year). The reason is that the number of active users in a given week, and the number of different programs they run, is a subset of the population that appears in the course of the year. To quantify this claim, we plotted the number of active users and the number of distinct applications or utilities submitted during each week (Figure 9). The number of active users varies from 26 to 43, out of a total of 88 users that were active during the whole period. The number of distinct applications varies from 56 to 180, out of a total of 1192 distinct applications executed during the whole

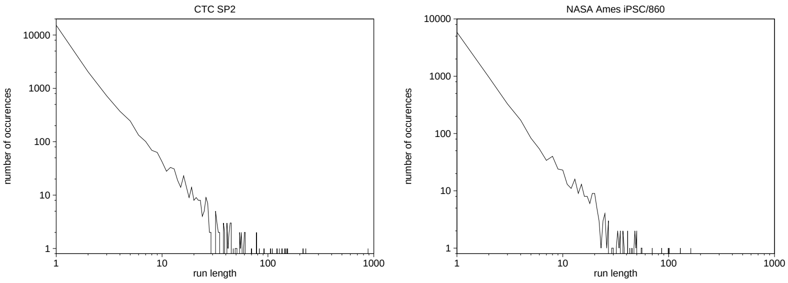


Figure 10: *Runlengths give an indication of the relative influence a single application may have on a workload. While most applications are only executed once or a small number of times, some are repeated hundreds of times.*

period.

Another reason to expect this kind of local predictability is that a single active user can make up a significant part of a workload over short periods of time. In general, many forms of human activity follow Zipf-like distributions: the most active person performs a certain amount of activity, the second most active performs half as much, the third most active a third, and so forth [39]. Thus comparing the workload when the most active user is on vacation with the workload when he is working on an important deadline might produce significantly different results.

An amazing example appears in the bottom row of Figure 7, where two weeks had a very large number of 128-node jobs (3662 and 1346). These observations are not a mistake; they represent 10 days of activity by a single user. These same jobs also caused the 14% peak in the top plot of Figure 4. While this is an extreme case, similar events are not uncommon, and the scheduler has to deal with them. Most of the other discrete peaks in Figure 4 can also be attributed to the activities of single (albeit less voracious) users [9].

The most active users typically achieve their mark by repeated execution of the same application on the same number of nodes, leading to local predictability and uniformity in the workload. Figure 10 shows histograms of the number of times various applications were run repeatedly (the run length) on two systems. While most applications are only run once or a small number of times, there are applications that are run hundreds of times in a row. As the resource requirements of the repeated runs tend to be similar, such long run lengths may create modes in the distributions.

Another way to describe these findings is that we are again faced with a long-tailed distribution — the distribution of user activity. Some users are much much more active than others, and bias the overall workload statistics towards their private workloads (which are often modal). The question is again whether to consider such outliers (like the user who ran some 5000 one-second 128-node jobs in 10 days) significant workload characteristics, or to ignore them.

6.2 Implications for workload modeling

Workloads have a certain *structure*: the pool of users varies over time; among the active users, a few tend to dominate the workload; these dominant users often run a few applications repeatedly. Thus while the overall variability of the workload may be high, the workload at any instant may be much more predictable. If we create a model that uniformly selects jobs with characteristics drawn from the proper global distributions, we will get the correct statistics, but will lose this structure.

This structure is relevant for two reasons. First, historical information about this structure may make it possible to predict the behavior of users and applications, allowing the scheduler to make better decisions. On the other hand, it may be more difficult to design schedulers to deal with this sort of workload, because strategies that do well on general workloads may be vulnerable to pathological behavior when the workload is less “random”.

One way to capture the structure of a workload is to model the arrival process along with other workload characteristics, perhaps by modeling the behavior of individual users. For example, Markov models have been proposed, whereby each user moves within a state space (representing the execution of different applications), and may remain in the same state for some time (representing the repeated execution of the same job) [4]. An alternative is to use a fractal model, such as those used to model network traffic or memory access patterns [27, 36].

6.3 The problem of sample size

For batch systems at supercomputer centers, even a year-long trace is rather small—it typically includes only tens of thousands of jobs, generated by a few tens of users. These traces are too small to yield "smooth" distributions, especially since the distributions typically have very large variability. Samples from short time spans are even more choppy.

Although ten thousand jobs may not sound like a small sample, it was recently shown that for certain non-preemptive schedulers the mean response time does not stabilize even after simulations of 300,000 jobs, due to short jobs that get delayed by very long jobs from the tail of the duration distribution [14]. Thus it may not be possible to produce a statistically valid evaluation of such strategies, even with a year-long trace.

This problem emphasizes the usefulness of workload models for generating synthetic traces, since synthetic traces can be arbitrarily long. Also, by using different random seeds, it is possible to generate a number of traces with identical statistical properties. Comparison of the performance of these traces is one way to check the statistical significance of a result.

Another way to check for errors due to sample size limitations is to run simulations over a range of time scales, and look for a relationship between the length of the simulation and the result. If there is such a relationship, it suggests that the result is an arbitrary artifact of the simulation rather than a meaningful statement about the system being evaluated.

7 Related and Future Work

The suggestion that workload modeling should be based on measurements is not new [15, 1]. However, relatively little work has been done on this subject. Examples include work on characterizing the distribution of durations [26, 17] and on the arrival process [3], both in the context of Unix workstations. In the area of parallel systems, descriptive studies of workloads have only started to appear in recent years [12, 38, 33, 9]. There are also some attempts at modeling [2, 7, 20] and on-line characterization [16].

Practically every section of this paper identifies topics that require additional research. One issue that was not addressed, however, is the relative importance of these topics. Is it really important to characterize the correlations between different attributes of workloads? Is it as important to capture the structure of the workload at different timescales? The answer is that importance depends on the effect each attribute has on the outcome of evaluations. This too requires additional research. Preliminary results indicate that in some cases simple models can get the qualitative results correctly, but more detailed models may be needed in order to obtain more quantitative results [30].

A chief concern in workload modeling is the need to distinguish between "real" workload characteristics and those that result from local policies. The problem is that users are humans with great adaptability, and the traces only record the final outcome—not the actual needs that drove the process. Indeed, one can't ignore the human aspect of workload generation, and including human behavior in the models is an intriguing possibility.

A recurring problem is the quality of trace logs. Low-quality traces may have internal inconsistencies or missing data for the same job (e.g. a start record and no end record, or vice versa). Moreover, traces typically contain no information on downtime and other special local circumstances. Thus it is necessary to choose traces carefully for modeling. For the future, one should encourage the maintenance of good logs, as they are the basis for all modeling work.

8 Conclusions

Constructing a model always involves tradeoffs between realism and complexity. The goal is to capture all relevant aspects of a real system while keeping the model as simple as possible. Until recently, workload models for parallel computers erred on the side of simplicity, yielding results that may not be applicable to real systems.

In this paper, we have surveyed recent efforts to construct more realistic workload models and discussed some of the issues that researchers in this area have addressed. We have enumerated the aspects of real workloads that we think are relevant to the evaluation of scheduling strategies and other system services for parallel computers.

Of course, not all of these aspects will be relevant to all models. For example, the performance of a scheduling strategy may depend strongly on daily variations in system load, but not on weekly variations. For some strategies, the frequency of jobs with power-of-two cluster sizes has a strong effect; for others it is irrelevant.

We hope that this paper will help researchers choose workload models that are appropriately simple, but realistic enough to yield applicable results. In addition, we hope it will spur additional research on models *per se*, on modeling methodology, and on the effect of models on evaluation results.

Appendix: Trace Data

The data used in this paper come from the following traces:

- A trace of all jobs executed during 1995 on the 416-node Intel Paragon installed at the San-Diego Supercomputing Center (SDSC). Many thanks to Reagan Moore and others at SDSC for making this data available.
- A trace of all jobs executed from January through September 1996 on the 1024-node Thinking Machine CM-5 installed at Los-Alamos National Lab (LANL). Many thanks to Curt Canada of LANL for making this data available.
- A trace of all batch jobs executed from September through November 1995, and January through April 1996, on the 512-node IBM SP2 installed at Cornell Theory Center (CTC). Many thanks to Steve Hotovy and others at CTC for making this data available.
- A trace of all jobs executed during the fourth quarter of 1993 on the 128-node Intel iPSC/860 hypercube installed at NASA Ames Research Center. Many thanks to Bill Nitzberg for making this data available.

These and other logs are available on-line from the parallel workloads archive web page, located at http://www.cs.huji.ac.il/labs/parallel/workload/.

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